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Decays of excited baryons in the large-N_c expansion of QCD

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Abstract. We present the analysis of the decay widths of excited baryons in the framework of the $1/N_c$ expansion of QCD. These studies are performed up to order $1/N_c$ and include both positive- and negative-parity excited baryons.

PACS. 11.15.Pg Expansions for large numbers of components (*e.g.*, $1/N_c$ expansions) – 13.30.-a Decays of baryons – 14.20.Gk Baryon resonances with $S=0$

1 Introduction

The $1/N_c$ expansion of QCD [1] has proven to be a useful tool for analyzing the baryon spectrum [2]. This success is mostly a consequence of the emergent contracted spin-flavor symmetry in the large-N_c limit [3,4]. Although spin-flavor is not an exact symmetry in the excited baryon sector [5], its N_c^0 breaking turns out to be small as several analyses of the baryon spectrum have shown [6–8]. We apply here the $1/N_c$ expansion of QCD to the study of the decays of the non-strange excited baryons belonging to the **70**-plet of negative parity and the **56**-plets of positive parity.

2 Strong decays

The ℓ_P partial-wave strong decay width of a resonance with total angular momentum, spin and isospin J^*, S^*, I^*, respectively, into a ground-state baryon with quantum numbers J, I and a pseudo-scalar meson with isospin I_P can be expressed as

$$\Gamma^{[\ell_P, I_P]} = \frac{k_P}{8\pi^2} \frac{M_B}{M_B^*} \frac{|B(\ell_P, I_P, J, I, J^*, I^*, S^*)|^2}{(2J^*+1)(2I^*+1)}, \quad (1)$$

where $B(\ell_P, I_P, J, I, J^*, I^*, S^*)$ are the reduced matrix elements of the baryonic operator. For each meson angular-momentum projection μ and isospin projection α, such

Table 1. Basis operators for the strong decays of the non-strange **70**-plet baryons. The operators for eta D-wave decays are not listed since no empirical information is available for those decays. nB indicates the n-bodyness of each operator.

		Name	Operator	Order
Pion S-wave	1B	$O_1^{[0,1]}$	$\left(\xi^{(1)}\ g\right)^{[0,1]}$	1
	2B	$O_2^{[0,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(s\ T_c)^{[1,1]}\right)^{[0,1]}$	$1/N_c$
		$O_3^{[0,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(t\ S_c)^{[1,1]}\right)^{[0,1]}$	$1/N_c$
		$O_4^{[0,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(g\ S_c)^{[1,1]}\right)^{[0,1]}$	$1/N_c$
Pion D-wave	1B	$O_1^{[2,1]}$	$\left(\xi^{(1)}\ g\right)^{[2,1]}_{[i,a]}$	1
	2B	$O_2^{[2,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(s\ T_c)^{[1,1]}\right)^{[2,1]}$	$1/N_c$
		$O_3^{[2,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(t\ S_c)^{[1,1]}\right)^{[2,1]}$	$1/N_c$
		$O_4^{[2,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(g\ S_c)^{[1,1]}\right)^{[2,1]}$	$1/N_c$
		$O_5^{[2,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(g\ S_c)^{[2,1]}\right)^{[2,1]}$	$1/N_c$
		$O_6^{[2,1]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(s\ G_c)^{[2,1]}\right)^{[2,1]}$	1
	3B	$O_7^{[2,1]}$	$\left(\frac{\xi^{(1)}}{N_c^2}\ \left(s\,(\{S_c, G_c\})^{[2,1]}\right)^{[2,1]}\right)^{[2,1]}$	$1/N_c$
		$O_8^{[2,1]}$	$\left(\frac{\xi^{(1)}}{N_c^2}\ \left(s\,(\{S_c, G_c\})^{[2,1]}\right)^{[3,1]}\right)^{[2,1]}$	$1/N_c$
eta S-wave	1B	$O_1^{[0,0]}$	$\left(\xi^{(1)}\ s\right)^{[0,0]}$	1
	2B	$O_2^{[0,0]}$	$\frac{1}{N_c}\ \left(\xi^{(1)}\,(s\ S_c)^{[1,0]}\right)^{[0,0]}$	$1/N_c$

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Table 2. Fit parameters. LO: there is two-fold ambiguity for the angle θ_1. For θ_3 there is an *almost* two-fold ambiguity given by the two values indicated in parenthesis and which only differ in the two slightly different values of $C_6^{[2,1]}$. NLO: due to lack of empirical data, 3B operators and the subleading operator for η emission are not included. No degeneracy in θ_1 but *almost* two-fold ambiguity in θ_3 given by the two values indicated in parenthesis. Values of coefficients which differ in the corresponding fits are indicated in parenthesis.

	LO	NLO
$C_1^{[0,1]}$	31 ± 3	23 ± 3
$C_2^{[0,1]}$	–	$(7.4, 32.5) \pm (27, 41)$
$C_3^{[0,1]}$	–	$(20.7, 26.8) \pm (12, 14)$
$C_4^{[0,1]}$	–	$(-26.3, -66.8) \pm (39, 65)$
$C_1^{[2,1]}$	4.6 ± 0.5	3.4 ± 0.3
$C_2^{[2,1]}$	–	-4.5 ± 2.4
$C_3^{[2,1]}$	–	$(-0.01, 0.08) \pm 2$
$C_4^{[2,1]}$	–	5.7 ± 4.0
$C_5^{[2,1]}$	–	3.0 ± 2.2
$C_6^{[2,1]}$	$(-1.86, -2.25) \pm 0.4$	-1.73 ± 0.26
$C_1^{[0,0]}$	11 ± 4	17 ± 4
θ_1	1.56 ± 0.15 0.35 ± 0.14	0.39 ± 0.11
θ_3	$(3.00, 2.44) \pm 0.07$	$(2.82, 2.38) \pm 0.11$
$\chi^2_{\rm dof}$	1.5	0.9
dof	10	3

operator admits an expansion in $1/N_c$ of the form [9,10]

$$B^{[\ell_P,I_P]}_{[\mu,\alpha]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_{q,j} C_q^{[\ell_P,I_P]} \left(\xi^{(\ell)}\ \mathcal{G}_q^{[j,I_P]}\right)^{[\ell_P,I_P]}_{[\mu,\alpha]}. \quad (2)$$

The factor $\left(\frac{k_P}{\Lambda}\right)^{\ell_P}$ is included to account for the chief meson momentum dependence of the partial wave, where we set the scale Λ to 200 MeV. The operator $\xi^{(\ell)}$ drives the transition from the $(2\ell+1)$-plet to the singlet $O(3)$ state. The operators $\mathcal{G}$ give transitions within the spin-flavor representations in which the excited and GS baryons reside. They can be written as products of the generators of the spin-flavor algebra acting on the excited quark state $\lambda = s_i, t_a, g_{ia}$ and on the core $\Lambda_c = (S_c)_i, (T_c)_a, (G_c)_{ia}$. The relevant operators for the non-strange members of the **70**-plet are given in table 1.

The dynamics of the decays is encoded in the effective dimensionless coefficients $C_q^{[\ell_P,I_P]}$. Here, they are obtained by fitting the available empirical decay widths [11]. In the case of the negative-parity baryons we have, in addition, two extra unknown parameters: the mixing angles θ_{2J} between the two sets of excited nucleon states N_J^*, where $J = 1/2, 3/2$. The results of the corresponding fits are shown in table 2. From those results we observe that, in general, the LO operators already provide a reasonable description of the decay widths. The NLO operators play some role in improving the results, although their coefficients are not well determined due to the large error bars in the empirical widths.

Table 3. Basis operators for the decays of the low-lying positive-parity baryons. Note that in the case of the $\ell = 0$ (Roper) baryons the operator $O_3^{[\ell_P,1]}$ is absent.

	Name	Operator	Order
1B	$O_1^{[\ell_P,1]}$	$\frac{1}{N_c}\left(\xi^{(\ell)}\ G\right)^{[\ell_P,1]}$	1
2B	$O_2^{[\ell_P,1]}$	$\frac{1}{N_c^2}\left(\xi^{(\ell)}\left([S\ ,\ G]\right)^{[1,1]}\right)^{[\ell_P,1]}$	$1/N_c$
	$O_3^{[\ell_P,1]}$	$\frac{1}{N_c^2}\left(\xi^{(\ell)}\left(\{S\ ,\ G\}\right)^{[2,1]}\right)^{[\ell_P,1]}$	$1/N_c$

Table 4. Fit parameters corresponding to the pion P-wave decays of $\ell = 0$ excited baryons.

Pion P-waves	LO	NLO
χ^2_{dof}	4.05	0.1
dof	3	2
$C_1^{[1,1]}$	18.7 ± 2.4	17.0 ± 1.6
$C_2^{[1,1]}$	–	24.4 ± 6.3

The basis operators for the decays of the positive-parity baryons are given in table 3. Note that, since the corresponding spin-flavor wave functions are completely symmetric, only the total generators $\Lambda = \lambda + \Lambda_c$ are required.

The results of the fits corresponding to the pion P-wave decays of $\ell = 0$ excited (Roper multiplet) baryons are given in table 4. We observe that in this case the LO fit leads to rather poor results. In fact, the NLO operator is essential to obtain a good description of these decays. It should be noted, however, that the coefficient of the LO order operator remains stable, and that the one of the NLO operator is of natural size.

The results for the decays of the $\ell = 2$ baryons are given in table 5. For the P-wave decays, the LO analysis already provides an excellent fit. In fact, the NLO analysis implies that the coefficients of the 2B operators are compatible with zero. In the case of the F-wave decays, the LO analysis gives a reasonable fit provided the empirical errors are taken to be 30% or more, which is the expected accuracy of such an analysis. If in the NLO fit the errors are taken at its empirical estimates the χ^2_{dof} turns out to be rather large. The main contribution to this quantity comes from the $\pi\Delta$ decay of the $N(1680)$ for which only an upper bound is given in ref. [11]. As shown in table 5, if such a decay is removed from the analysis a quite good fit is obtained.

Another interesting observation that points to the consistency of the framework based on the approximate $O(3) \times SU(2N_f)$ symmetry is that the predicted

Table 5. Fit parameters corresponding to the pion P- and F-wave decays of $\ell = 2$ excited baryons. In the case of the LO fit of the F-wave decays those empirical errors that are less than 30% have been increased up to that value. In the corresponding NLO fit the empirical value of the decay $N(1680) \to \pi\ \Delta$ has not been considered.

Pion P-waves	LO	NLO
χ^2_{dof}	0.15	0.44
dof	3	1
$C_1^{[1,1]}$	6.83 ± 0.77	4.09 ± 0.47
$C_2^{[1,1]}$	–	0.11 ± 2.41
$C_3^{[1,1]}$	–	0.43 ± 6.09
Pion F-waves	LO	NLO
χ^2_{dof}	1.73	0.38
dof	4	1
$C_1^{[3,1]}$	1.01 ± 0.09	0.84 ± 0.04
$C_2^{[3,1]}$	–	-1.30 ± 0.21
$C_3^{[3,1]}$	–	-0.26 ± 0.47

suppression of the η channels in the decays of positive-parity baryons is clearly displayed by the observed decays. This represents a strong experimental confirmation that those baryons do belong primarily to a symmetric representation of $SU(2N_f)$. In the case of the negative-parity baryons the η channel is not suppressed and is indeed very important. This implies that these states belong primarily to the mixed-symmetric representation of $SU(2N_f)$. Thus, η channels serve as a selector of the spin-flavor structure of excited baryons.

3 Radiative decays

The situation concerning the radiative decays is in a more preliminary stage. In ref. [12] a LO analysis of the radiative decays of the $(\mathbf{70}, 1^-)$-plet states has been performed, and a reasonable fit obtained. Perhaps the most important conclusion of that study is that, although there are already 2B operators contributing to this order, the amplitudes are dominated by 1B operators. Of course, to have a clearer understanding a NLO analysis is required. Concerning the radiative decays of the positive-parity resonances belonging to the **56**-plet, a study which includes up to NLO corrections is currently under way [13]. Preliminary results indicate that, as in the case of the strong decays, the LO analysis seem to be problematic for Roper baryons. On the other hand, at least some LO model independent relations seems to work well for the $(\mathbf{56}, 2^+)$-plet states. For example, for the decays of the $\Delta(1950)$ one obtains the model-independent relation $A^N_{3/2}/A^N_{1/2} = 1.29$ to be compared with the empirical value 1.27 ± 0.24.

4 Conclusions

To conclude, the $1/N_c$ expansion provides a systematic approach to the properties of the excited baryons. The analysis of the masses shows that the N_c^0 breaking of the spin-flavor symmetry is small. For strong decays one finds, in general, a dominance of the one-body LO operators. In some cases, as *e.g.* the D-wave decays of the negative-parity excited baryons, the $1/N_c$ corrections are not well established due to the rather large uncertainties of the empirical data. In the particular case of the Roper baryons, two-body NLO operators seem to be required to obtain a good description of the empirical decay widths. In the case of the radiative decays, there is work in progress. Preliminary results indicate that the situation is similar to that of strong decays. Finally, we should notice that, here, we have followed the so-called operator approach to excited baryons in large-N_c QCD. Since the decay widths of such baryons are of order N_c^0, there is also an alternative large-N_c description of such states as resonances in meson-baryon scattering [14].

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